# A MANAGEMENT MODEL FOR THE CENTRAL STOCK OF THE NORTHERN ANCHOVY, ENGRAULIS MORDAX 

JOHN RADOVICH<br>California Department of Fish and Game Sacramento, CA 95814

ALEC D. MACCALL
California Department of Fish and Game
Southwest Fisheries Center
La Jolla, CA 92038


#### Abstract

The average growth of the northern anchovy central subpopulation between the years 1951 and 1975, as measured by CalCOFI egg and larva surveys, can be empirically described by a logistic growth curve. The asymptotic maximum population size, or spawning biomass, is 4 million short tons, and the intrinsic rate of increase is 0.36 . A maximum average growth rate of 360,000 short tons/year occurs at a spawning biomass of 2 million tons. If the population were maintained at 2 million tons by harvesting surplus production, an estimated 450,000 short tons/year could be harvested. The difference between yield and population growth rate is due to the competing risk of death between fishing mortality and natural mortality. This estimate of potential productivity is smaller than previous estimates, but is better founded.


## INTRODUCTION

In the absence of a substantial fishery, there has been some difficulty in getting good estimates of potential yield of the anchovy population. For most years, the catch was less than $5 \%$ of the biomass (MacCall et al. 1976), certainly too small a fishery to be able to distinguish fishery-caused effects from other variations (including sampling error).

In the early 1950's, Clark and Phillips (1952) indicated that the anchovy fishery resource probably could not withstand a major fishery at that time. By the late 1950's, as a result of CalCOFI investigations, it had become obvious that the anchovy population had grown, and a potential harvest was believed to be possible. In 1964, Chapman (1964), referred to a harvest rate of $25 \%$ of the population, which he felt was a very conservative estimate of the potential harvest. Subsequently, the $25 \%$ value was frequently stated as a conservative potential catch of the recruited biomass. It had no biological basis and was viewed with skepticism by a number of scientists.

No further progress was made on estimating maximum sustainable yield (MSY) until MacCall et al. (1974, 1976) applied a technique that has come to be known as Gulland's "quick and dirty" potential yield method (Gulland 1970). This method roughly estimates potential yield from a fishery in which most fishery data is lacking and where the catch is an insignificant portion of the biomass. The Gulland Formula is:

$$
\begin{aligned}
& Y_{\text {pot }}= X M B_{\mathrm{o}} \\
& \text { where } \begin{aligned}
Y_{\text {pot }}= & \text { potential yield; } \\
M= & \text { instantaneous natural mortality rate; } \\
B_{\mathrm{O}}= & \text { mean virgin biomass of fish above length } \\
& \text { at first capture; } \\
X= & \text { a coefficient which is determined by } M, K \\
& \text { (von Bertalanffy growth rate }) \text {, and rela- } \\
& \text { tive size at first capture }\left(l_{c}\right) .
\end{aligned}
\end{aligned}
$$

Biomass estimates from egg-and-larva surveys were used since they appeared to be the most reliable source. These correspond to the spawning biomass which may not differ substantially from the exploitable biomass since the fish appear to be recruited and mature at one year of age (Table 1). However, anchovies become available to the fishery at about the time of first spawning, so spawning biomass as measured by egg and larva surveys must therefore represent initial rather than average spawning biomass for the season. Thus, the biomass estimates used by MacCall et al. were larger than mean biomass, and potential yield was similarly overestimated.

A total mortality of $Z=1.1$ was calculated from catch curves for five different years (MacCall 1974). $F$ was obtained from a ratio of the catch to total biomass estimates. These averaged 0.03 . This indicates that $M$ cannot be smaller than 1.00 or 1.05 . They used calculated values of 0.3 for Bertalanffy growth parameter $K$ and 165.5 mm (standard length) for von Bertalanffy parameter $L_{\infty}$. Since recruitment occurs at over a range of 85 to $115 \mathrm{~mm}, 105$ mm was used as the estimate of $l_{c}$. Considering the stock very near virgin state, Gulland's equation $Y_{\text {pot }}=X M B_{\mathrm{o}}$ was used to estimate potential yield using an $M$ of 1.05 and $K$ of 0.3 , an $l_{c}$ of 105.0 mm , and an $L_{\infty}$ of 165.5 mm ,

$$
\frac{M}{K}=\frac{1.05}{0.3}=3.5 \text { and } c=\frac{l_{c}}{L_{\infty}}=\frac{105.0}{165.5}=0.63 .
$$

Using the values for $M / K$ and $c$, the value of the coefficient $X$ would be about 0.6 , from Gulland's table (Gulland 1970, p. 3). The maximum potential yield should therefore be about $60 \%$ of the mean fishable biomass. From this, the potential yields from 1951 through

[^0]TABLE 1
Anchovy Biomass Estimates for the Central Subpopulation, and Annual Catches (Thousand Short Tons).

| Year | Biomass* | Catch |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | U.S. |  | Mexico | Total |
|  |  | Commercial | Live Bait |  |  |
| 1951. | 180 | 3 | 5 | ..... | 8 |
| 1952. | 156 | 28 | 7 | ..... | 35 |
| 1953 | 510 | 43 | 6 | $\ldots$ | 49 |
| 1954 | 768 | 21 | 7 | $\ldots$ | 28 |
| 1955. | 846 | 22 | 6 | $\ldots$ | 28 |
| 1956. | 485 | 28 | 6 | ..... | 34 |
| 1957. | 1,172 | 20 | 4 | ..... | 24 |
| 1958. | 1,479 | 6 | 4 | ..... | 10 |
| 1959. | 1,514 | 4 | 5 | ..... | 9 |
| 1960 | 1,540 | 2 | 5 | ..... | 7 |
| 1961 | 1,159 | 4 | 6 | $\ldots$ | 10 |
| 1962. | 2,986 | 1 | 6 | 1 | 8 |
| 1963 | 4,254 | 2 | 4 | 1 | 7 |
| 1964. | 2,900 | 2 | 5 | 5 | 12 |
| 1965 | 4,659 | 3 | 6 | 10 | 19 |
| 1966 | 3,572 | 31 | 7 | 15 | 53 |
| 1967. |  | 35 | 5 | 22 | 62 |
| 1968 |  | 15 | 7 | 16 | 38 |
| 1969 | 2,999 | 68 | 5 | 4 | 77 |
| 1970. |  | 96 | 6 | 31 | 133 |
| 1971. |  | 45 | 6 | 22 | 73 |
| 1972. | 2,784 | 69 | 6 | 36 | 111 |
| 1973. |  | 133 | 6 | 16 | 155 |
| 1974. |  | 83 | 6 | 44 | 133 |
| 1975. | 3,603 | 159 | 6 | 61 | 225 |

*Biomass estimates supplied by Paul Smith (NMFS). These estimates are in the process of being revised and are preliminary.

TABLE 2
Biomass and Corresponding Potential Yields for the Total Population and the Central Stock for 1) the 20-year Period of Observation and 2) after the Depletion of Pacific Sardine Stock.*

| Time period | Population | Biomass mean and range ( $10^{3}$ tons) | Corresponding potential yields ( $10^{3}$ tons) |
| :---: | :---: | :---: | :---: |
| 1951-69 | Total | $\begin{gathered} 3,200 \\ (640-7,800) \end{gathered}$ | $\begin{gathered} 1,900 \\ (400-4,700) \end{gathered}$ |
| 1951-1972 after <br> After sardine collapse | Central stock | $\begin{gathered} 2,600 \\ (290-6,200) \end{gathered}$ | $\begin{gathered} 1,500 \\ (200-3,700) \end{gathered}$ |
| 1965-69 .... | Total | $\begin{gathered} 4,600 \\ (2,200-7,800) \end{gathered}$ | $\begin{gathered} 2,800 \\ (1,300-4,700) \end{gathered}$ |
| 1965-72 | Central stock | $\begin{gathered} 4,000 \\ (2.050-6.200) \\ \hline \end{gathered}$ | $\begin{gathered} 2,400 \\ (1,200-3,700) \end{gathered}$ |

*After MacCall et al. (1976).
1972 were calculated (Table 2). These averaged from 1.5 to 2.4 million tons for the central subpopulation, although a very wide range was given. MacCall et al. concluded that the central stock of northern anchovy can produce a yield from 10 to 20 times higher than present catches. In view of the numerous collapses of clupeid fisheries and the known low abundances of anchovies in the recent past, their statement appeared extreme. While such levels of harvest might well be feasible for a short
while, their sustainability appeared questionable.
The anchovy fishery is being subjected to strong pressure for expansion. The Mexican fleet is unilaterally developing their anchovy reduction industry. Passage of the Fisheries Management and Conservation Act of 1976 by the U.S. Legislature portends a likely increased harvest, if not by U.S. fishermen, then possibly by foreign fleets under the provisions of the Act. A more reliable estimate of sustainable yield of anchovy is needed. This estimate would preferably be based on observed dynamics of the northern anchovy central stock rather than on broad generalizations based on characteristics of different species in different environments, as is the Gulland method.

In 1975 we noticed that since 1950 the anchovy population seemed to have at first grown slowly, then accelerated rapidly, and finally leveled off somewhere around 4 million tons. This pattern was very similar to the logistic growth curve. The low abundance of anchovy in 1951 has not been attributed to any demonstrable cause. The species was not harvested to any extent, ruling out overexploitation. The hypothesis of interspecific competition from the Pacific sardine (Sardinops sagax caerulea) has been invoked (Murphy 1966), seemingly verified (Silliman 1969), and set back (Soutar and Isaacs 1974). Smith (1972) showed the anchovy population to have been relatively large (between 2 and 3 million tons) in 1940 and 1941. Such a population size at a time when sardines were also abundant is utterly inconsistent with the competition hypothesis. It appears much more likely that the anchovy suffered a sequence of recruitment failures probably near the end of the 1940 's. Two other local pelagic species, the Pacific sardine (MacCall 1979) and the Pacific mackerel, Scomber japonicus (Parrish 1974; Parrish and MacCall 1978), showed severe reproductive failures in the period 1949-1951. A third species, the Pa cific bonito (Collins and MacCall 1977), became very scarce in California waters during the early 1950's. The anchovy may well have suffered similar reproductive difficulties.
Following this decline, after 1951, the central stock first increased its density at the center of its distribution in southern California waters. Then, with the warm ocean temperatures of the late 1950 's, the central stock extended its range to the waters of central California, which had originally been inhabited by an apparently local population. Since 1960 the density of fish in the occupied areas increased, reaching relative stability in the late 1960's and early 1970's.

If the low abundance in 1951 was due to some limiting factor, expansion of the central stock can be interpreted as an increase in carrying capacity relative to that prevailing in the early 1950's. If the low abundance was due to a long series of poor year-classes because of environmental variability and the anchovy subsequently
experienced a long series of favorable reproductive conditions, the present level of abundance could be very temporary, with no level of fishing necessarily being sustainable. Until the actual cause of the increase is determined, the possibility of a "natural" decline to a lower population must be be considered in any management strategy. However, growth rates of the population during that favorable period provide a basis for estimating reasonable interim harvest rates, and the interim period may continue for an indefinite period of time. The sedimentary scale records of Soutar and Isaacs (1974) indicate that low anchovy populations are a rare event.

## METHODS

A logistic growth curve is based on the assumption that at very low population sizes the population growth rate is limited by the small biomass present, while at large population sizes the growth rate is reduced because all suitable habitat, food, or other resource necessary for existence is utilized, allowing no further expansion. Specific population growth rate $(d B / B d t)$ decreases linearly with the fraction of the maximum carrying capacity that is utilized. Maximum growth rate occurs at a population size that is exactly halfway between zero and the maximum capacity.

Various fisheries models, termed "production models," utilize this concept, wherein the fishery controls the population size so as to hold it at a level optimal for fish production. If the yield from the fishery matches the potential increase, an equilibrium is reached. The highest equilibrium point is called maximum equilibrium yield or maximum sustainable yield (MSY). A drawback with such fisheries models is that they must use a proxy (or substitute) for the population estimate by assuming a relationship between effort or catch per effort and the actual population. This may introduce serious biases as each of the authors of this paper have suggested previously (Radovich 1973, 1975, 1976; MacCall 1976). However, with the anchovy, we are in a unique situation of having fishery-independent estimates of population size of the stock during a 25 -year period of considerable population growth.

If we make the assumption that in 1950 the anchovy stock was very low (for whatever cause or restraining influence) and subsequently grew in logistic fashion to the maximum carrying capacity of the geographical area, due to the removal of the initial restraining influence, we can obtain essentially the same management information that usually is attempted to be derived from a fishery by the use of production models.

The CalCOFI spawning biomass estimates for the central subpopulation of northern anchovy (Table 1, Figure 1) are assumed to represent the biomass in the spring of each year, which is the time of peak spawning activity.

These data were used to estimate the parameters of the logistic growth equation.

$$
B_{(t)}=\frac{B_{\infty}}{1+e^{-r\left(t-t_{O}\right)}}
$$

where: $B_{(t)}$ is the biomass at time $t$,
$r$ is the intrinsic rate of increase,
$B_{\infty}$ is the asymptotic maximum biomass, or carrying capacity, and
$t_{o}$ is the time of inflection of the growth curve (time at which $B_{(t)}=1 / 2 B_{\infty}$ ).

The biomass data were first transformed by taking logarithms to obtain a constant error variance. They were then submitted to a curvilinear least squares regression algorithm (Conway et al. 1970), which employed the corresponding log transform of the above logistic equation. Due to $\log$ transformation, parameter $B_{\infty}$ was estimated as a geometric mean, so the correction of Beauchamp and Olson (1973) was applied to obtain the proper arithmetic mean. The resulting parameter estimates were

$$
\begin{aligned}
& B_{\infty}=4.0 \text { million tons }, \\
& r=0.36, \\
& t_{0}=1959.6,
\end{aligned}
$$

and the curve is plotted in Figure 1.


Figure 1. A logistic growth curve fitted to the spawning population biomass estimates of the central stock of the northern anchovy from 1951 to 1975.

The first time derivative of the logistic growth equation gives population growth rate as a function of biomass:

$$
\frac{d B_{(t)}}{d t}=r B \frac{B_{\infty}-B}{B_{\infty}}
$$

This equation describes growth rate as a domed curve rising from the origin to a peak growth rate at $1 / 2 B_{\infty}$ (Figure 2). Since there is considerable variation about the regression line in Figure 1, this estimate of growth rate must be interpreted as an average (itself subject to error of estimation) about which actual observations will vary.


Figure 2. Differential form of the growth curve (dashed line) and yield curve (solid line) of the northern anchovy.

If a harvest is taken each year such that all the expected growth is caught immediately before the biomass is measured, the biomass would tend to remain unchanged from measurement to measurement, and equilibrium yield would be obtained. In this manner, the growth rate curve can be interpreted as an equilibrium yield curve. However, if the catch were made just after measurement of biomass, somewhat more fish could be harvested while maintaining equilibrium, since many are likely to die of natural causes before the next biomass measurement. For an anchovy fishery acting continuously throughout the year, equilibrium yield would be greater than the corresponding growth potential by perhaps $25 \%$. The actual percentage would vary with natural mortality rate and with the seasonality of the fishery. Thus, for the northern anchovy central stock, we estimate the peak growth rate would be 360,000 tons/year at a spawning biomass of 2.0 million tons, whereas the equilibrium yield at the same biomass would be closer to 450,000 tons/year (Figure 2), which is our estimate of maximum sustainable yield.

## DISCUSSION

This use of the logistic growth model violates several assumptions that normally underlie the logistic equation. However, many of these assumptions are simply dispensed with by treating the logistic as an empirical de-
scription of the population growth curve. We need not be concerned with the mechanisms by which logistic growth is usually assumed to occur.

Nontheless some assumptions can bear serious discussion. By using the logistic curve as a regression line, about which observations vary, the original deterministic nature of the curve is lost. Therefore, the growth rate curve and derived production curve now represent average responses, about which we expect variation from year to year. This implies that equilibrium biomass may not be welldefined under any particular fishing strategy, and many years of observations may be necessary to determine the true population response to fishing pressure.

The model also assumes that no fishing occurred during the period from which the data came. A moderate fishery existed for a few years in the early 1950's and again since the mid-1960's. Since the population was low in the early 1950's, the earlier fishery probably contributed to a downward bias in our estimate of the intrinsic rate of increase, whereas the later fishery caused a downward bias of perhaps 0.05 million tons (the magnitude of the fishery) in $B_{\infty}$. Finally, if the increase in carrying capacity were not immediate in 1951, but increased in stepwise fashion, the estimate of $r$ is likely to be low. This increase in carrying capacity may have occurred by expanding the geographic range of the stock. On the other hand, if the geographic range of the stock shrinks under exploitation (which has been observed in other fisheries), the production curve may be realistic.

At large biomasses, considerable amounts of fish in excess of MSY can be harvested, but these harvests are temporary and will only serve to bring the biomass down to a level where theoretical maximum sustained production would occur. Various harvest strategies can be drawn in the diagram (Figure 3). The line rising from the center to the upper right on the management diagram represents the harvest that will bring the population to optimum size ${ }^{2}$ in the very next year. This line, combined with the production curve, divides the diagram into four sectors. In Sector I the population will usually increase, and it will be above the optimal the following year. If a harvest enters Sector III, the population will usually increase, but it will not usually reach the optimum size the following year. At large population sizes, harvests in Sector II will take advantage of the biomass in excess of that needed for maximum growth. In Sector IV the population is being overfished; it will usually be declining and will be less than optimum the following year.
A steep harvest line will allow maximum utilization of excess biomass but would result in large year-to-year changes in catch, which could create problems with the economics of the fishing industry. A more horizontal

[^1]harvest line would tend to stabilize annual fluctuations. In drawing in such a harvest line, some errors will be compensated, and optimal harvest will be likely. For example,


Figure 3. The yield curve divided into four sectors by a line representing the locus of catches that would result in an optimum size population at the beginning of the following season.
consider a harvest line which is a diagonal straght line passing from 0 harvest at $1 / 2$ optimum biomass through MSY at optimum biomass and extending upward with larger biomasses (Figure 4). If the true harvestable production of the population is larger than we have shown, then such a harvest line would result in the fishery coming into an equilibrium at a highler level of catch but also at a higher biomass. Thus, the danger of overfishing the population is small, since large harvests would be allowed only when the population is very large; if the population can sustain such largest harvests, it will do so without danger of seriously depleting the stock. If biomass itself contributes to the objectives of management, it may be appropriate to shift the intersection of the harvest policy toward maintenance of larger biomass at a small reduction in yield. For example, the second alternative policy in Figure 4 would result in a gain of 320,000 tons of biomass while only 12,000 tons of yield would be lost relative to the first alternative policy.

An important feature of this harvesting strategy is the limit of biomass below which no harvest is allowed. This is a virtual guarantee that the resource cannot be depleted by overfishing. It also allows a maximal rate of recovery in the event of a natural decline in biomass as is assumed to have happened in the late 1940's. Finally, it recognizes the importance of the anchovy as a major trophic link in the food web. Although there is undoubtedly room for harvest of anchovies from the ecosystem, it would be unwise to place the importance of that harvest above the maintenance of the ecosystem that produced it (as well as


Figure 4. Alternate fishing strategies. The dshed line represents a policy that would allow for an average biomass 320,000 tons larger than that represented by the solid line, with an average loss in yield of 12.000 tons.
a wealth of higher predators).
The problem of the effects of the fishery on bait and recreational species has not been resolved; however, it is interesting to note that the heaviest runs of subtropical sportfish in northern California occurred in the mid-tolate 1950's (Radovich 1961). This took place when the biomass of anchovies was growing the most rapidly. It therefore appears that, under favorable environmental conditions, forage at the optimum population size, $B_{\infty} / 2$, was adequate to support an immigration of voracious game fish, as well as forage for the resident demersal fishes in southern California and for the live bait requirements of the recreational fishing fleet. It is important to note that under unfavorable environmental conditions, catches of migratory predator fish may be poor despite an abundance of anchovies. Many factors appear to influence migrations and abundance of predators, and much remains to be learned about their interactions. In addition, there does not appear to have been any known increase of other predators on anchovies, such as birds, since the anchovy biomass increased from its mid-1950's level.

When we compare this estimate of productivity with those made by the Gulland potential yield method, the Gulland estimates appear to be high. A similar discrepancy was shown by MacCall (1979) for the Pacific sardine fishery, on the basis of simulations. Besides the previously discussed error of confusing egg and larva biomass estimates with mean biomass, use of the Gulland method appears to assume an underlying stability of the resource that may not hold. In any case, our estimate of productivity, like the Gulland potential yield estimate, is not based on actual fishing experience and is meant to be modified as further information is gained.

## SUMMARY

In summary then, this method of estimating anchovy productivity has a number of advantages over other methods:

1. It is based on observed growth rates instead of assumed rates.
2. It does not rely on fishery catch and effort data and thereby avoids the serious biases encountered in the use and interpretation of effort measurements.
3. It gives a far more cautious estimate of MSY (about 450,000 tons), which seems more consistent with other studies, than Gulland's "quick and dirty" methods.
4. Management strategy should be directed toward maintaining an optimum spawning biomass, which may result in a more variable fishery than MSY obtained by other methods, but would not run as large a risk of collapse.

The minimum of the logistic curve (Figure 1) at zero and the carrying capacity at 4 million tons can be fit easily, but the slope at the point of inflection isn't quite as clear. This means that the maximum equilibrium catch may be somewhat higher or lower depending on a few points which may vary quite a bit; however, the optimum population size still appears to be about one-half the carrying capacity, and the management strategy of attempting to keep the population at its optimum size is valid. One may simply proceed with caution and watch where the population tends to stabilize, before making further adjustments.

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[^0]:    'The von Bertalanffy growth curve is $l_{t}=L_{\infty}\left(1-e^{-K\left(t-t_{0}\right)}\right)$ where $l_{t}$, is length at time $t . L_{\infty}$ is asymptotic maximum length. $K$ is a growth constant and $t_{0}$ is hypothetical age at zero length.

[^1]:    "Optimum" as used here refers to that population sive producing maximum yield and does not imply any other kind of optimality

