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THE HYDROSTATIC EQUATION AND SIMPLE CALCULATION OF HEIGHT

A hydrostatic equilibrium exists when there is no vertical acceleration in the atmosphere. Basic equation of hydrostatics says then that a heigth difference *dz* corresponding to a small vertical pressure difference *dp* (e.g. 1 Pa) can be derived from

$$dp = -g\rho dz \qquad (1)$$

where g is the acceleration of gravity and ρ is the air density at this height. Negative sign results from that the height (z) increases upwards and the pressure (p) increases downwards.

When calculating the heigth difference ($h = z - z_0$) between two pressure surfaces (e.g. p_0 = 1000 hPa and p = 500 hPa) the average air density between these surfaces needs to be known. As measuring it is quite complicated and the density in the formula (1) can be replaced with temperature T derived from the ideal gas law resulting

$$dz = -\frac{RT}{gp}dp \qquad (2)$$

where R is the general gas constant. By integrating both sides of the equation (2) the height difference (or thickness of the air layer) is

$$h = \left(\frac{R}{g}\right)\langle T\rangle \ln\left(\frac{p_0}{p}\right) \qquad (3)$$

where $\langle T\rangle$ is the average temperature in the layer $p_0 - p$.

<u>Example:</u>
p = 995 hPa, p_0 = 1000 hPa, t = 24 °C, t_0 = 25 °C => T = 297.15 °K and T_0 = 298.15 °K. Constants: R = 287.05 J/Kg°K and g = 9.80665 m/s^2.

From (3)

$$h = \left(\frac{287.05}{9.80665}\right)\left(\frac{297.15 + 298.15}{2}\right)\ln\left(\frac{1000}{995}\right) = 43.7m.$$

Literature:
John M. Wallace, Peter V. Hobbs: Atmospheric Science, an Introductory Survey, Academic Press (1977)